The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2020-2021

MECHANICS OF SOLIDS

Time allowed TWO Hours plus 30 minutes upload period

Open-book take-home examination

Answer ALL questions

You must submit a single pdf document, produced in accordance with the guidelines provided on take-home examinations, that contains all of the work that you wish to have marked for this open-book examination. Your submission file should be named in the format '[Student ID]_MMME2053.pdf'.

Write your student ID number at the top of each page of your answers.

This work must be carried out and submitted as described on the Moodle page for this module. All work must be submitted via Moodle by the submission deadline. **Work submitted after the deadline will not be accepted without a valid EC.**

No academic enquiries will be answered by staff and no amendments to papers will be issued during the examination. If you believe there is a misprint, note it in your submission but answer the question as written.
Contact your Module Teams Channel or SS-AssessEng-UPE@exmail.nottingham.ac.uk for support as indicated in your training.

Plagiarism, false authorship and collusion are serious academic offences as defined in the University's Academic Misconduct Policy and will be dealt with in accordance with the University's Academic Misconduct Procedures. The work submitted by students must be their own and you must declare that you understand the meaning of academic misconduct and have not engaged in it during the production of your work.

ADDITIONAL MATERIAL: Formula sheet

SECTION A

1. Calculate the position of the centroid, C, for the beam cross-section shown in Fig. Q1.

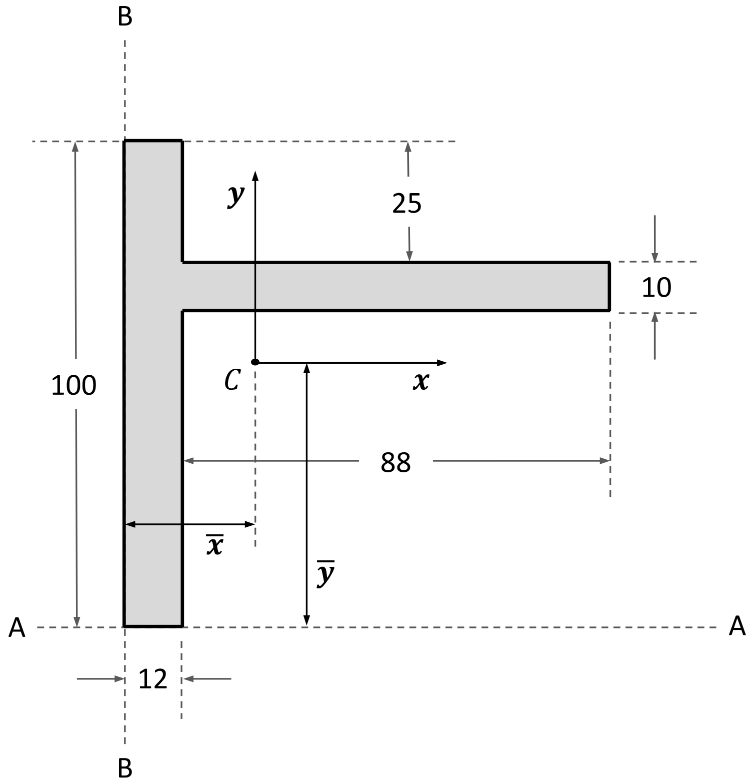

Fig. Q1

A. $\bar{x} = 27.15\ \mathrm{mm},\ \bar{y} = 41.54\ \mathrm{mm}$
B. $\bar{x} = 82.85\ \mathrm{mm},\ \bar{y} = 0\ \mathrm{mm}$
C. $\bar{x} = 27.15\ \mathrm{mm},\ \bar{y} = 58.46\ \mathrm{mm}$
D. $\bar{x} = 82.85\ \mathrm{mm},\ \bar{y} = 41.54\ \mathrm{mm}$
E. $\bar{x} = 0\ \mathrm{mm},\ \bar{y} = 0\ \mathrm{mm}$

[2]

2. Using the parallel axis theorem, what is the product moment of area, $I_{x'y'}$, for the section shown in Fig. Q1?

A. $1{,}851{,}524.13\ \mathrm{mm}^4$
B. $507{,}692.32\ \mathrm{mm}^4$
C. $1{,}210{,}410.26\ \mathrm{mm}^4$
D. $0\ \mathrm{mm}^4$
E. $507{,}827.68\ \mathrm{mm}^4$

[2]

3. The following expression describes the strain energy in a beam:

$$U = \frac{P^3}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right) + \frac{Q^2}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$$

What is the deflection at the position of and in the direction of the applied load, P?

A. $u_P = \frac{P^4}{4EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right)$

B. $U_P = \frac{P^3}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right) + \frac{Q^2}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

C. $u_P = \frac{3P^2}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right) + \frac{Q^2}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

D. $u_P = \frac{3P^2}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right)$

E. $U_P = \frac{2Q}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

[2]

4. What is the deflection at the position of and in the direction of the dummy load, Q in the beam from Q3?

A. $u_Q = \frac{2Q}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

B. $U_Q = \frac{3P^2}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right)$

C. $u_Q = \frac{Q^3}{3EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

D. $u_Q = \frac{P^3}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right) + \frac{2Q}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

E. $U_Q = \frac{P^3}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + 2\pi R^2\right) + \frac{Q^2}{EI}\left(\frac{L}{3} + \frac{3\pi R}{8}\right)$

[2]

5. For a beam for which an expression for bending moment, M, can be derived, and deflection is measured as positive upwards:

A. $EI\frac{d^2y}{dx^2} = M$

B. $EI\frac{dy}{dx} = M$

C. $EI\frac{d^2y}{dx^2} = -M$

D. $EI\frac{d^2y}{dx^2} = M^2$

E. $EIy = -M$

[2]

6. Decreasing the length of a beam which is loaded along its length:

A. Has no effect on the load required to cause buckling
B. Means the beam will definitely buckle
C. Eliminates the concern of buckling
D. Reduces the load required to cause buckling
E. Increases the load required to cause buckling

[2]

7. Two cylinders of equal length have the following dimensions:

Cylinder 1 (C1): $12\ \mathrm{mm}$ bore and $24.03\ \mathrm{mm}$ outside diameter (steel: $E = 210\ \mathrm{GPa}$, $\nu = 0.3$, $\alpha = 12 \times 10^{-6}\ °\mathrm{C}^{-1}$)

Cylinder 2 (C2): $24\ \mathrm{mm}$ bore and $44\ \mathrm{mm}$ outside diameter (bronze: $E = 100\ \mathrm{GPa}$, $\nu = 0.3$, $\alpha = 17 \times 10^{-6}\ °\mathrm{C}^{-1}$)

If Cylinder 2 is heated and placed around Cylinder 1 and allowed to cool, what are the boundary conditions required to solve the stress distributions using thick cylinder theory?

A. C1-ID $\sigma_r = 0$, C1-OD $\sigma_r = -p$, C2-ID $\sigma_r = p$, C2-OD $\sigma_r = 0$

B. C1-ID $\sigma_r = 0$, C1-OD $\sigma_r = 0$, C2-ID $\sigma_r = 0$, C2-OD $\sigma_r = 0$

C. C1-ID $\sigma_r = 0$, C1-OD $\sigma_r = p$, C2-ID $\sigma_r = -p$, C2-OD $\sigma_r = 0$

D. C1-ID $\sigma_r = 0$, C1-OD $\sigma_r = -p$, C2-ID $\sigma_r = -p$, C2-OD $\sigma_r = 0$

E. C1-ID $\sigma_r = 0$, C1-OD $\sigma_r = p$, C2-ID $\sigma_r = p$, C2-OD $\sigma_r = 0$

[2]

8. If the I-section beam shown in Fig. Q8 is subjected to a vertical shear force of $50\ \mathrm{kN}$, what is the value of the maximum horizontal shear stress in the flange?

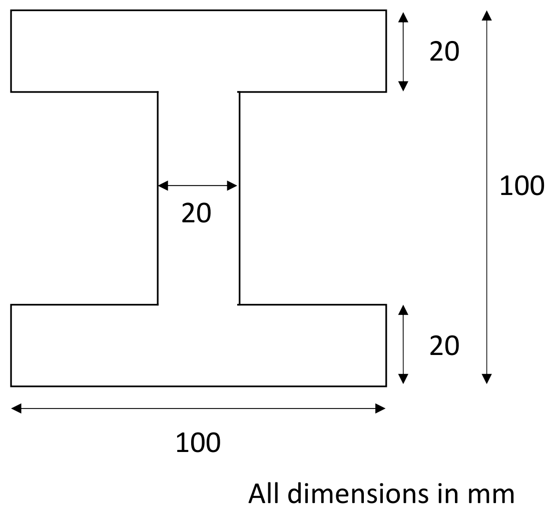

Fig. Q8

A. $12.0\ \mathrm{MPa}$
B. $7.5\ \mathrm{MPa}$
C. $2.4\ \mathrm{MPa}$
D. $24.0\ \mathrm{MPa}$
E. $14.6\ \mathrm{MPa}$

[2]

9. A bar with a rectangular cross-section $(b \times d)$ of $30\ \mathrm{mm} \times 35\ \mathrm{mm}$ subjected to a vertical shear force of $15\ \mathrm{kN}$, what is the value of maximum shear stress in the bar?

A. $50.0\ \mathrm{MPa}$
B. $21.4\ \mathrm{MPa}$
C. $18.4\ \mathrm{MPa}$
D. $25.0\ \mathrm{MPa}$
E. $29.2\ \mathrm{MPa}$

[2]

10. A $600\ \mathrm{mm}$ long, 1D bar element has a rectangular cross section of $30\ \mathrm{mm} \times 40\ \mathrm{mm}$ and is made of a material with a Young's Modulus of $210\ \mathrm{GPa}$, the stiffness matrix of the element is:

A. $[k_{el}] = 2.52 \times 10^8 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

B. $[k_{el}] = 1.3 \times 10^{-3} \begin{bmatrix} 2.52 \times 10^8 & -2.52 \times 10^8 \\ -2.52 \times 10^8 & 2.52 \times 10^8 \end{bmatrix}$

C. $[k_{el}] = \begin{bmatrix} 210 \times 10^9 & -210 \times 10^9 \\ -210 \times 10^9 & 210 \times 10^9 \end{bmatrix}$

D. $[k_{el}] = 2.52 \times 10^8 \begin{bmatrix} 1.667 & -1.667 \\ -1.667 & 1.667 \end{bmatrix}$

E. $[k_{el}] = \begin{bmatrix} 2.52 \times 10^8 & -2.52 \times 10^8 \\ -2.52 \times 10^8 & 2.52 \times 10^8 \end{bmatrix}$

[2]

11. A shaft, made of a material with $\sigma_y = 315\ \mathrm{MPa}$, will carry a torque of $21\ \mathrm{kNm}$. According to the Tresca yield criterion, what should the radius be to avoid yielding?

A. $26\ \mathrm{mm}$
B. $44\ \mathrm{mm}$
C. $43\ \mathrm{mm}$
D. $35\ \mathrm{mm}$
E. $33\ \mathrm{mm}$

[2]

12. A large steel plate containing a crack, for which $K_I = 1.85\sigma\sqrt{\pi a}$, has a fracture toughness, $K_{I_{cr}}$, of $170\ \mathrm{MPa}\sqrt{\mathrm{m}}$ and a yield stress, σ_y, of $235\ \mathrm{MPa}$. If the applied stress, σ, is $\frac{3}{4}\sigma_y$, determine the critical crack size assuming linear elastic material.

A. $165\ \mathrm{mm}$
B. $854\ \mathrm{mm}$
C. $86.5\ \mathrm{mm}$
D. $271.8\ \mathrm{mm}$
E. $48.7\ \mathrm{mm}$

[2]

13. In the design of a new service component, elastic-perfectly-plastic material behaviour is:

A. More conservative than linear hardening
B. Less conservative than non-linear hardening
C. The only option
D. Less conservative than linear hardening
E. Equal to linear hardening in terms of conservatism

[2]

14. What is the value of the equivalent (von Mises) stress for the 2D plane-stress element shown in Fig. Q14?

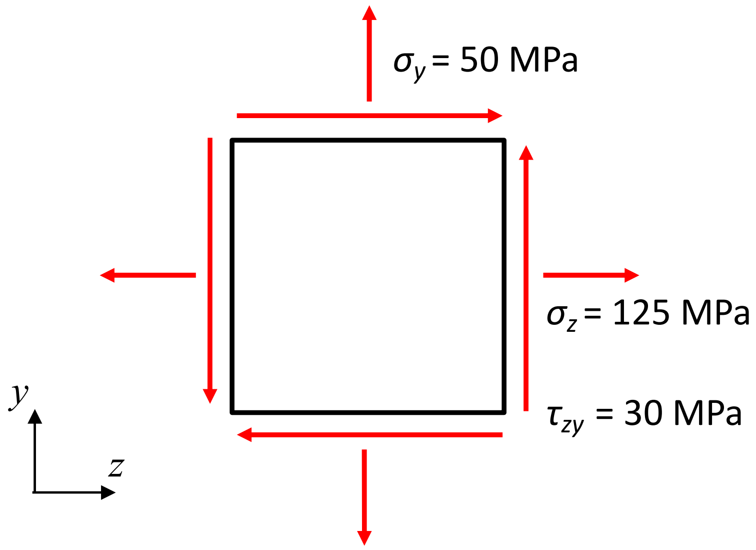

Fig. Q14

A. 48.0 MPa
B. 120.7 MPa
C. 135.5 MPa
D. 39.5 MPa
E. 87.5 MPa

[2]

15. A solid $30\ \mathrm{mm}$ diameter bar is subjected to a torque of $0.75\ \mathrm{kNm}$ and a pure bending moment of $1.25\ \mathrm{kNm}$, the maximum principal stress on a 2D plane stress element on the surface of the bar is:

A. 471.6 MPa
B. 141.5 MPa
C. 275.0 MPa
D. 235.8 MPa
E. 510.8 MPa

[2]

16. A thick-walled cylinder of ID $40\,\mathrm{mm}$ and OD $100\,\mathrm{mm}$ is subjected to an internal pressure of $145\,\mathrm{bar}$ and an external pressure of $50\,\mathrm{bar}$, what is the value of hoop strain on the surface of the cylinder? $E = 208\,\mathrm{GPa}$ and $\nu = 0.33$.

A. 0.1457×10^{-4}
B. 0.3106×10^{-4}
C. 0.013×10^{-4}
D. 0.274×10^{-4}
E. 0.4692×10^{-4}

[2]

17. In order to increase conservatism for design against fatigue:

A. R-ratio should be increased
B. Maximum load makes no difference
C. R-ratio should be decreased
D. R-ratio makes no difference
E. Minimum load should be zero

[2]

18. For a hinged-fixed strut of length, L, under compressive loading, comprehensive set of boundary conditions are:

A. (i) $x = 0, y = 0$, (ii) $x = L, y = 0$ & (iii) $x = L, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$
B. (i) $x = 0, y = 0$ & (ii) $x = L, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$
C. (i) $x = 0, y = 0$, (ii) $x = 0, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ & (iii) $x = L, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$
D. (i) $x = 0, y = 0$ & (ii) $x = L, y = 0$
E. (i) $x = 0, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ & (ii) $x = L, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$

[2]

19. A circular cross-section beam is required to have a 2nd moment of area of at least $200{,}000{,}000\,\mathrm{mm}^4$. What is the minimum possible diameter of the beam?

A. $126.3\,\mathrm{mm}$
B. $212.5\,\mathrm{mm}$
C. $166.3\,\mathrm{mm}$
D. $336.4\,\mathrm{mm}$
E. $252.6\,\mathrm{mm}$

[2]

20. A shaft, made of a material with $\sigma_y = 275\ \mathrm{MPa}$, will carry a torque of $27\ \mathrm{kNm}$. According to the von Mises yield criterion, what should the radius be to avoid yielding?

A. $10.4\ \mathrm{mm}$
B. $102.7\ \mathrm{mm}$
C. $69.8\ \mathrm{mm}$
D. $47.7\ \mathrm{mm}$
E. $18.4\ \mathrm{mm}$

[2]

SECTION B

21. A straight "sandwich" beam, symmetrical about the Y-Y axis, is made of two different materials. The cross section is shown in Fig. Q21. The beam will be deemed unsafe if any plasticity occurs in material B.

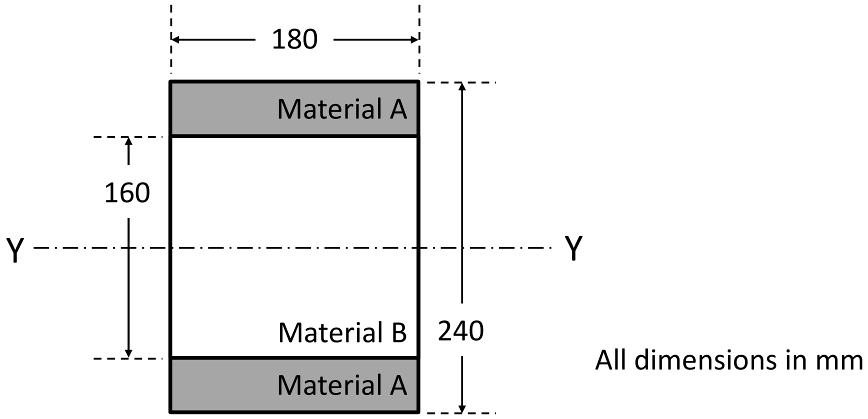

Fig. Q21

(a) After application of a pure bending moment, $M = 540\ \mathrm{kNm}$ about the Y-Y axis, the beam is deemed unsafe for the reason stated above. Show by calculation, that this is true, and determine the distance, a, from the Y-Y axis, that plasticity occurs. [12]

(b) Sketch the residual stress state in the beam when the bending moment is removed. [8]

Both materials A and B, can be assumed to be elastic-perfectly-plastic with yield stresses $\sigma_{y_A} = 200\ \mathrm{MPa}$ and $\sigma_{y_B} = 190\ \mathrm{MPa}$, respectively.

22. A beam is simply supported at both ends, as shown in Fig. Q22. It is subjected to a point moment, M_O, and a uniformly distributed load, q, as shown.

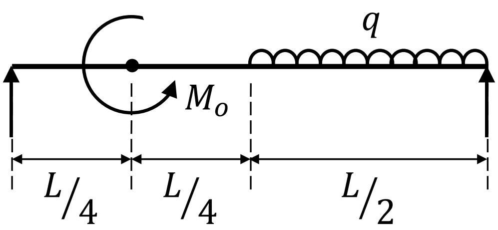

Fig. Q22

Derive expressions, in terms of E, I, M_O, q and L, for the slope and deflection at any position in the beam and evaluate for both slope and deflection at the position of the applied point moment and at the mid-point of the beam. [20]

23. A steel cylinder with an internal diameter of $120\ \mathrm{mm}$, an external diameter of $160\ \mathrm{mm}$ and a length of $1250\ \mathrm{mm}$, is restrained between two walls at P and Q, as shown in Fig. Q23. Initially the cylinder is at $T_0 = 20\ °\mathrm{C}$ and there is no initial axial force. Assume $\alpha = 11 \times 10^{-6}\ °\mathrm{C}^{-1}$ and $E = 208\ \mathrm{GPa}$ for aluminium.

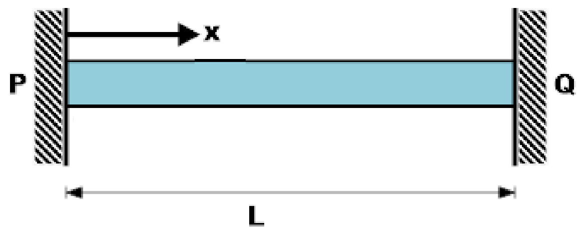

Fig. Q23

(a) Determine the resultant axial force and therefore stress in the cylinder if the temperature is increased to $75\ °\mathrm{C}$. [2]

(b) Determine the resultant axial force and therefore stress in the cylinder if the temperature distribution along the length of the cylinder is given by:

$$T = T_0 + \frac{55x}{L}$$

[8]

(c) If the cylinder is then subjected to an internal pressure of $120\ \mathrm{bar}$ and an external pressure of $25\ \mathrm{bar}$ in addition to the temperature gradient of (b), determine the value of hoop stress and therefore von Mises stress on the surface of the cylinder at $x = \frac{L}{2}$, ignoring any end effects. [10]